

Using the notation in the Mondal and Kusmznovic paper, say rate of flow i is $1/(2^{k_i} RTO_i)$. Average loss rate on link of rate C carrying n flows is $p = (\sum_{j=1}^n 1/(2^{k_j} RTO_j) - C) / \sum_{j=1}^n 1/(2^{k_j} RTO_j)$. Assume same loss rate is seen by all flows (this seems like a strong assumption that needs to be validated). Mean goodput for flow i is then

$$\frac{1-p}{2^{k_i} RTO_i}$$

and mean per packet completion time is

$$T_i = \frac{2^{k_i} RTO_i}{1-p} = \frac{2^{k_i} RTO_i \sum_{j=1}^n 1/(2^{k_j} RTO_j)}{C}$$

Question 1: We ask whether it is true that

$$T_i \geq \frac{RTO_i \sum_{j=1}^n 1/RTO_j}{C}$$

i.e. the T_i is always at least as great as the completion time when $k_i = 1 \forall i \in \{1, \dots, n\}$.

A simple counter-example that answers this question in the negative is as follows. Letting the number of flows $n = 2$,

$$T_1 = \frac{1 + (2^{k_1}/2^{k_2})(RTO_1/RTO_2)}{C}$$

while

$$\frac{RTO_1(1/RTO_1 + 1/RTO_2)}{C} = \frac{1 + RTO_1/RTO_2}{C}$$

Evidently, whenever $k_1 < k_2$ we have that $T_1 < \frac{RTO_1 \sum_{i=1}^n 1/RTO_i}{C}$.

Question 2: We ask whether it is true that

$$\sum_{i=1}^n T_j \geq \sum_{i=1}^n \frac{RTO_i \sum_{j=1}^n 1/RTO_j}{C}$$

i.e. is the aggregate flow completion time with $k_i \geq 1$ always above the value when $k_i = 1 \forall i$.

$$\sum_{i=1}^n T_i = \frac{1}{C} \sum_{i=1}^n (2^{k_i} RTO_i) \sum_{j=1}^n 1/(2^{k_j} RTO_j)$$

A similar counter-example that answers this question in the negative is as follows. Let the number of flows $n = 2$, and let $RTO_2 = 2^\alpha RTO_1$. Then

$$T_1 + T_1 = \frac{2 + 2^{k_1}/2^{k_2} RTO_1/RTO_2 + 2^{k_2}/2^{k_1} RTO_2/RTO_1}{C} = \frac{2 + 2^{k_1}/2^{(k_2+\alpha)} + 2^{(k_2+\alpha)}/2^{k_1}}{C}$$

while

$$\frac{(RTO_1 + RTO_2)(1/RTO_1 + 1/RTO_2)}{C} = \frac{2 + 1/2^\alpha + 2^\alpha}{C}$$

When $k_1 = k_2 + 1$, $\alpha = 1$ then

$$2^{k_1}/2^{(k_2+\alpha)} + 2^{(k_2+\alpha)}/2^{k_1} = 2 < 1/2^\alpha + 2^\alpha = 2\frac{1}{2}$$

and so

$$\sum_{i=1}^n T_j < \sum_{i=1}^n \frac{RTO_i \sum_{j=1}^n 1/RTO_j}{C}$$